MEASURING THE HEIGHT OF A MOUNTAIN WITH TRIANGLES

Purpose

To measure the height of a mountain by the method of triangulation

Resources Needed

Brunton pocket transit
Measuring chain (100-foot steel tape measure)
GPS (Global Positioning System) receiver (Garmin eTrex 20 or equivalent)
Topographic map

Preparation

Do an Internet search for "Stone Mountain Topography" (I use "duckduckgo"; use Google if you like)

Select TopoQuest (https://www.topoquest.com/map-detail.php?usgs_cell_id=43306)

Scroll down to Stone Mountain Park Police Station. Select it.

The website in the browser address bar is: https://www.topoquest.com/map.php?lat=33.80477&lon=-84.14747&datum=nad27&zoom=4&map=auto&coord=d&mode=pan&size=m

Click on the map. Scroll down to the center of the map.

Place your cursor on "Beacon" on top of the mountain. Drag upward and leftward about 1 cm to center the mountain in the map viewer. Note the Covered Bridge on the right (east) side of the map. Be sure not to drag the map too far. You must be able to see the Covered Bridge on the right.

The road from the Covered Bridge ends at a crossroad that encircles the mountain. Place your cursor on the intersection where the Covered Bridge road ends. Do not drag the map, but just move the cursor to the left about half a cm until the green color ends. This is the end of the tree line in the East Quarry and is near the location of the start of my baseline. If you know how to read a contour map, you will see that the elevation of this location falls about midway between the 900 and 920 foot elevation contours. The GPS verifies the elevation of this location as 910 feet above mean sea level. The baseline extends about 550 feet downward toward the SSE (south-south-east), staying to the right (east) of the 920-foot contour.

Background

Stone Mountain is an igneous monadnock (quartz monzonite isolated dome-shaped mountain) that rises 786 feet above the 900-foot elevation contour (the accepted base level of the mountain). It is surrounded by trees at its base and rises very steeply on three sides. The fourth side rises steeply near the top. This makes it impossible to see the top of the mountain when standing near its base. A television transmitting antenna is located on top of the mountain near its highest point. The top of the antenna extends 346 feet above the summit of the mountain and serves as a good reference for measuring the height of the mountain using basic surveying techniques. This antenna is vaguely visible in the upper left corner of the photo at the right.

First-Approach Surveying Tool

The most portable surveying tool for geologic mapping is the Brunton pocket transit. This instrument is capable of measuring horizontal angles (*compass bearings*, or *azimuths*) to a *precision* of 0.5° and vertical angles to a precision of 10 arcminutes, or 0.17°. The *working precision* for this project is 0.5° in both the vertical and the horizontal directions. A photograph of this transit is shown on Page 2. The circular scale of numbers around the outer edge of the transit represents compass bearings. The innermost semicircular scale of numbers is used to measure vertical angles in degrees. These are known as elevation or altitude angles. The outermost semicircular scale of numbers represents percent gradient. One hundred percent gradient equals 45° of arc. The smaller scale between the innermost and outermost scales is a vernier scale that allows altitude precisions down to 10 arcminutes. The "bull's eye" bubble level inside the transit on the upper left is used to level the transit for bearing measurements, while the straight bubble level above center is used to measure altitude angles. Normal precisions for linear scales are generally defined as one half the smallest scale division. The north-arrow pointer on the right is capable of distinguishing integer degrees and half degrees (one half the smallest scale division). The mirror on the left and the pointer on the right are used in tandem to measure "back azimuths." *Back azimuths* are reverse azimuths useful in geologic mapping.

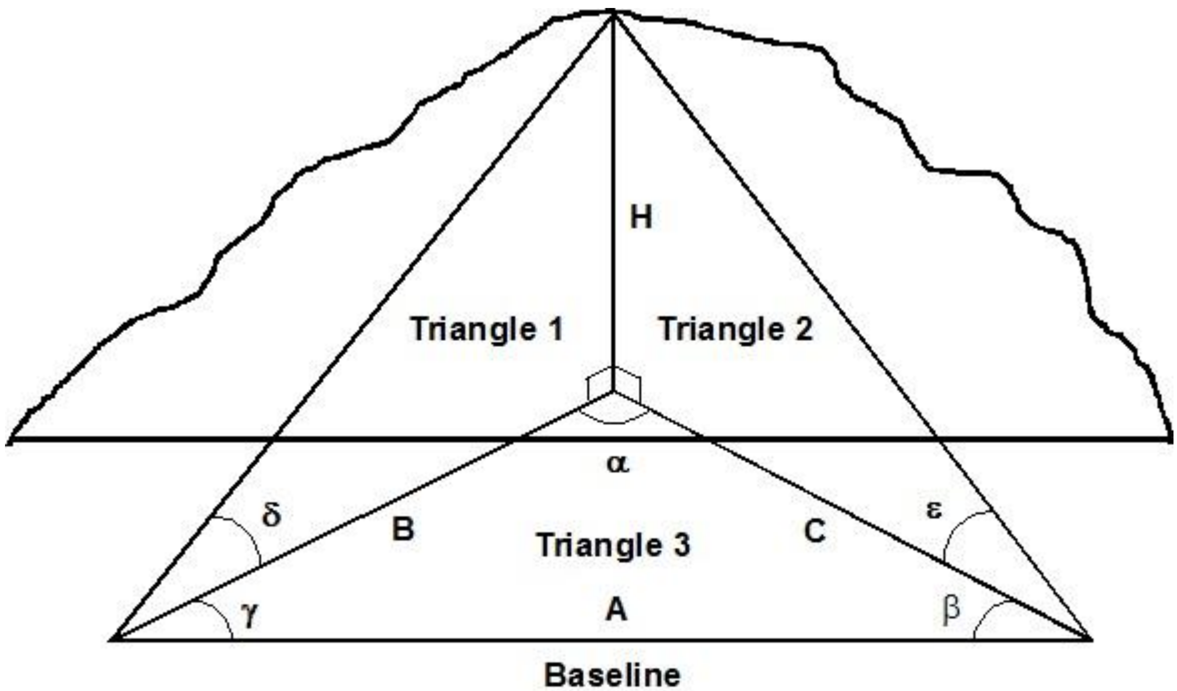
Types of Measurement

Angular measurements, linear distance measurements, and GPS altitudes are needed for this surveying project. Since USGS (United States Geological Survey) topographic maps are scaled in units of feet, these are the distance units that will be used.

Linear distance measurements are made according to two methods: pacing and chain. Every student who takes a rigorous structural geology course in college is required during laboratory hours to determine the length of his/her pace. One pace equals two steps forward. Pace length is determined by counting the number of steps taken in crossing a known distance (e.g., 100 feet). The pace is calibrated by walking speed. The faster the speed, the longer the pace. The slower the speed, the shorter the pace. After traversing the distance dozens of times over a period of several days, the proper speed can be determined that will consistently allow a pace length of 5 feet. This is the pace length used when measuring linear distances in the project by pace. The chain method makes use of a 100-foot steel tape measure referred to as a *chain*, from the days when actual chains were used.

Geometric Layout

Below is a quick sketch that was drawn for this project using the MS Word drawing tool.



The baseline is a line of reference to which all other measurements correspond. The altitude of the baseline above mean sea level will be referred to as *ground level*. Triangle 1 is a vertical right triangle that extends along side B from the left end of the baseline horizontally to a location vertically below the top of the mountain at the altitude of the baseline (ground level), then vertically upward to the top of the mountain, and back again as the hypotenuse of this right triangle. The angle δ is a vertical angle that subtends side B and the hypotenuse of Triangle 1, thereby making side B the adjacent side and side H the opposite side. Triangle 2 corresponds to Triangle1, but is located at the right end of the end of the baseline. The angle ε is a vertical angle that subtends side C and the hypotenuse of Triangle 2, thereby making side C the adjacent side and side H the opposite side. Triangle 3 is a scalene horizontal triangle in which side A is the baseline, side B is the base of Triangle1, and side C is the base of Triangle 2. All sides of Triangle 3 are at ground level. Note that Triangle 3 is solved with the *law of sines*.

The Measurements

Horizontal angle β subtends sides A and C of Triangle 3. This angle is determined by two bearing (compass direction) measurements: one along side A and the other along side C. The difference in these two bearings is angle β. Horizontal angle γ subtends sides A and B of Triangle 3. It is determined by two bearing measurements: one along side A and the other along side B. The difference in these two bearings is angle γ. Horizontal angle α is calculated by subtracting angles β and γ from 180°. The length of A is the only linear distance measured.

Measurement Uncertainties

All physical measurements are subject to *measurement uncertainty*. This uncertainty is often referred to as *measurement error*. Quantities derived from calculations involving two or more measurements manifest an uncertainty that is related to the uncertainty of each of the measurements. Estimation of the uncertainties of these derived quantities is determined by methods referred to as *propagation of errors*. Error propagation is based on statistical models and differential calculus. Simplified methods of propagating errors are taught in college-level physics laboratories. In general, when a derived quantity is arrived at by the operations of addition or subtraction, the error in this quantity is found by addition of the *absolute error* in each of the contributing measurements. When a derived quantity is arrived at by the operations of multiplication or division, the error in this quantity is found by addition of the *relative error* in each of the contributing measurements. The absolute error is the uncertainty associated with a given measurement. For example, as described on Page 1, the working precision of angle measurements made by the Brunton pocket transit is 0.5°. This working precision is also a quantification of the measurement uncertainty. Thus the uncertainty, or absolute error, in each of the project angle measurements is 0.5°. If the measured angle is, for example, 25°, then the relative error for this measurement is found by dividing the absolute error by the quantity measured.

$$\text{Relative error} = \frac{\text{absolute error}}{\text{measurement}} = \frac{0.5^\circ}{25^\circ} = 0.02 = 2\%$$

Mathematical Relations

Triangle 1
$$\tan\delta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{H}{B} \rightarrow H = B\tan\delta$$

Triangle 2
$$\tan\varepsilon = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{H}{C} \rightarrow H = C\tan\varepsilon$$

Triangle 3
$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} \rightarrow B = A \times \frac{\sin\beta}{\sin\alpha}$$

$$\frac{\sin\alpha}{A} = \frac{\sin\gamma}{C} \rightarrow C = A \times \frac{\sin\gamma}{\sin\alpha}$$

Data Entry

Bearing from right end of baseline to left end (B1): ________________

Bearing from right end of baseline to top of mountain (B2): ________________

Elevation angle from right end of baseline to top of mountain (ε): ________________

Length of baseline (side A of Triangle 3): ________________ ft

Bearing from left end of baseline to right end (B3): ________________

Bearing from left end of baseline to top of mountain (B4): ________________

Elevation angle from left end of baseline to top of mountain (δ): ________________

Comments About Measurements

Calculations

Note that the top of the mountain cannot be seen from ground level. Therefore the altitude measurements are made to the top of the tall antenna installed near the top of the mountain, rather than to the top of the mountain itself. The height of the antenna above the mountain summit must be subtracted from the height measurements made from the ground. A1 and A2 below represent the calculated heights of the top of the antenna above ground level. The summit of the mountain is 786 feet above the 900-foot elevation contour, but only 776 feet above the baseline ground level of 910 feet. Note also that in standard propagation of errors, the error terms are summed in quadrature. See below.

β = B2 – B1 = ___________ – ___________ = ___________

γ = B3 – B4 = ___________ – ___________ = ___________

α = 180° – β – γ = 180° – ___________ – ___________ = ___________

$$B = A \times \frac{\sin\beta}{\sin\alpha} = \frac{\sin \text{\_\_\_\_\_\_\_\_\_\_}}{\sin \text{\_\_\_\_\_\_\_\_\_\_}} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ft}$$

$$C = A \times \frac{\sin\gamma}{\sin\alpha} = \frac{\sin \text{\_\_\_\_\_\_\_\_\_\_}}{\sin \text{\_\_\_\_\_\_\_\_\_\_}} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ft}$$

A1 = B × tan δ = ______________ × tan ___________ = ______________ ft

A2 = C × tan ε = ______________ × tan ___________ = ______________ ft

H1 = A1 – 346 ft = ______________ ft – 346 ft = ______________ ft

H2 = A2 – 346 ft = ______________ ft – 346 ft = ______________ ft

The error in H1 and H2 is 8% each.

Error (E1) in H1 = 8% × H1 = ___________ ft Error (E2) in H2 = 8% × H2 = ___________ ft

Error in average of heights H1 and H1 (E): $\sqrt{(E1)^2 + (E2)^2} = \sqrt{(\text{\_\_\_\_\_\_\_\_})^2 + (\text{\_\_\_\_\_\_\_\_})^2}$ = __________ ft

Average height of mountain (Z): (H1 + H2) ÷ 2 = (__________ + __________) ÷ 2 = ______________ ft

Average height of mountain ± error limits: Z ± E = ______________ ± __________ ft

The average height of the mountain with error limits shows the *quality* of the measurement.

Lower limit of measured height: Z – E = __________ ft Upper limit of measured height: Z + E = __________ ft

Lower limit ≤ 776 ft ≤ Upper limit → __________ ft ≤ 776 ft ≤ __________ ft